# Growth, Income Distribution and Policy Implications of Automation

Manoj Atolia, Morgan Holland and Jonathan Kreamer

Florida State University
University of Wyoming

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Research and Computing Center, FSU

# Automation: the question

- Growing public concern about **automation**.
  - Labor-substituting technological progress.
- Concerns about **distributional consequences**.
  - Growing inequality? Declining labor share?
- Implications for **long-run growth**.
  - What happens when all tasks can be automated?
  - Can this even happen? Under what conditions?
- Discussion of **policy responses**.
  - Proposals for Universal Basic Income (UBI).
  - Or other transfer programs (need-based; industry-specific).

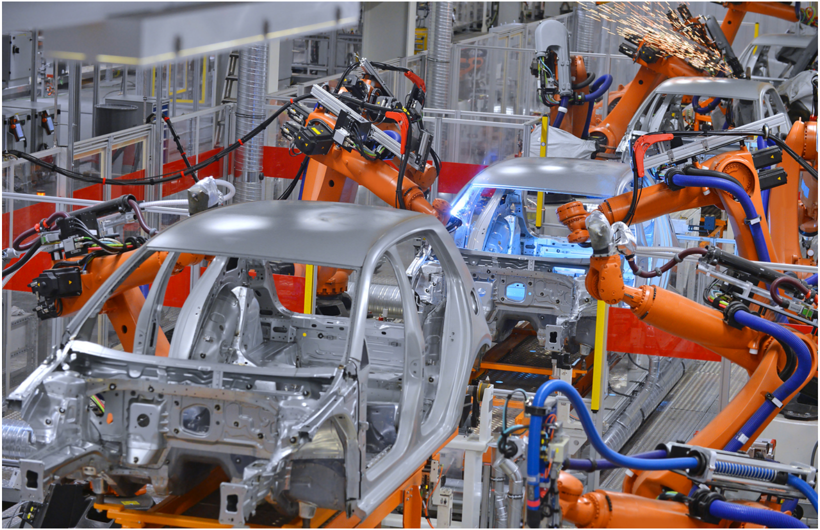

Figure: Robots on Assembly Line

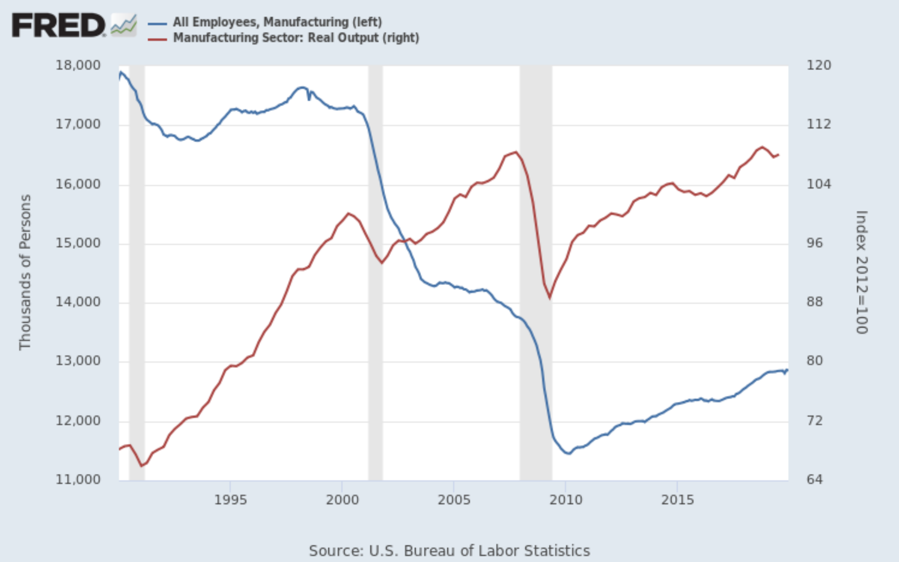


Figure: Manufacturing Employment and Output

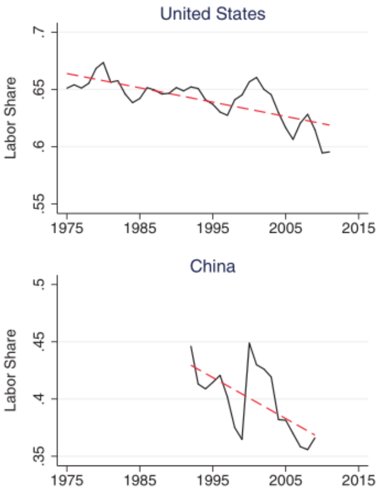


FIGURE II

Declining Labor Share for the Largest Countries

figure shows the labor share and its linear trend for the four largest economies in the world from 1975.

Figure: Labor share (Karabarbounis & Neiman 2014)

# Our approach

**Model:**

- Task-based model of automation.
  - Tasks can be done by labor or capital.
- Entrepreneurs and workers.
  - Focus on distributional implications.

**Analysis:**

- Examine consequence of an automation episode.
  - Possibility of complete automation.
  - Implications for income shares.
- Then look at political economy implications.
  - Worker-dominated government.
  - Characterize policy in response to automation episode.

# Literature

- Empirical results:
  - Task/skill-biased technical change (Autor et al 2003, Acemoglu Autor 2011).
  - Recent decline in labor share (Karabarbounis & Neiman 2014, Autor & Salomons 2018).
- Also several theoretical models of automation:
  - Acemoglu and Restrepo (2018), Aghion, Jones, Jones (2019).
  - Korinek Stiglitz (2019 book chapter) focuses on distribution
  - Prettner (2019) looks at growth.
- Optimal capital taxation (Judd 1985, Chamley 1986, Lansing 1999, Straub Werning 2020).

# Model

- Continuous time. Suppress time arguments for convenience.
- Two kinds of households: workers and entrepreneurs.
- **Workers**: cannot own capital; supply labor. Preferences:

$$\int_0^\infty e^{-\gamma t} U\left(C_w, L\right) dt$$

- Consumption and labor supply:

$$C_w = \left(1 - \tau^\ell\right) wL + T_w$$

$$-U_L\left(C_w, L\right) \leq \left(1 - \tau^\ell\right) wU_C\left(C_w, L\right)$$

# Workers & Entrepreneurs

- **Entrepreneuers**: own capital. Choose investment. Preferences:

$$\int_0^\infty e^{-\rho t} u\left(c_e\right) dt$$

- Assume $\rho \leq \gamma$ (entrepreneurs relatively patient). Decisions:

$$\dot{K} + c_e = r^k K, \quad \text{where } r^k = \left(1 - \tau^k\right) r - \delta$$

$$\frac{\dot{c}_e}{c_e} = \frac{r^k - \rho}{\varphi}, \quad \text{where } \varphi = -\frac{u''\left(c_e\right) \cdot c_e}{u'\left(c_e\right)}$$

# Production

- CES production technology (related to Acemoglu and Restrepo 2018):

$$Y = \left[ \int_0^1 (y(i))^{1-\frac{1}{\sigma}} \, di \right]^{\frac{\sigma}{\sigma-1}}$$

- Task $i$ can be performed by capital or human labor:

$$y(i) = a(i) k(i) + b(i) \ell(i)$$

- Assumption: $a(i)/b(i)$ weakly decreasing in $i$; $k(i), \ell(i) \geq 0$.
  - Implies cutoff task $\alpha$, s.t. tasks $i \leq \alpha$ done by capital, $i > \alpha$ by labor.

# Aggregate Representation of Production (Prop. 1)

- Under optimal production plan, output is:

$$Y(K, L, \alpha) = \left[\alpha^{\frac{1}{\sigma}} \left(A(\alpha)K\right)^{1-\frac{1}{\sigma}} + (1-\alpha)^{\frac{1}{\sigma}} \left(B(\alpha)L\right)^{1-\frac{1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}$$

- where $K = \int_0^\alpha k(i)\, di$, $L = \int_\alpha^1 \ell(i)\, di$, and:

$$A(\alpha) = \left[\frac{1}{\alpha} \int_0^\alpha (a(i))^{\sigma-1}\, di\right]^{\frac{1}{\sigma-1}}, B(\alpha) = \left[\frac{1}{1-\alpha} \int_\alpha^1 (b(i))^{\sigma-1}\, di\right]^{\frac{1}{\sigma-1}}$$

- and where $\alpha$ is implicitly defined by:

$$\begin{cases} \frac{a(i)}{b(i)} \geq q(\alpha, K, L) & i < \alpha \\ \frac{a(i)}{b(i)} \leq q(\alpha, K, L) & i > \alpha \end{cases} \qquad q(\alpha, K, L) = \frac{F_K}{F_L} = \frac{r}{w}$$

# Aggregate Representation of Production

- Under optimal production plan, output is:

$$Y = \left[\alpha^{\frac{1}{\sigma}} (AK)^{1-\frac{1}{\sigma}} + (1-\alpha)^{\frac{1}{\sigma}} (BL)^{1-\frac{1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}$$

- Technical change can have two effects:
  1. **Traditional technological progress:** increase $A$. Intensive margin.
  2. **Automation:** increase $\alpha$. Extensive margin.

# Government

- The government budget constraint:

$$T_w = \tau^{\ell} wL + \tau^{k} rK - G$$

- For now, assume:
  - Fixed tax rates $(\tau^{\ell}, \tau^{k})$.
  - Zero government spending $G = 0$.
- Transfer to workers may change over time as $wL$ and $rK$ change.

# Existence of Steady State vs. Sustained Growth

- **First result**: steady state may not exist!
  - Possible to have sustained growth through capital accumulation alone.
  - Call this **full automation** scenario.
- Intuition: as $L/K \to 0$, production function approaches $AK$.
  - If "A" sufficiently high, continuous growth occurs.
- Under full automation:
  - Labor share goes to $0$.
  - Generally $w > 0$. $L > 0$ or $L = 0$ both possible.

# Existence of Steady State... (Prop. 2)

## Proposition

*Let:*

$$A(1) = \left[ \int_0^1 (a(i))^{\sigma-1} \, di \right]^{\frac{1}{\sigma-1}}$$

$$r^* = \frac{\rho + \delta}{1 - \tau^k}$$

*Then:*

1. *If* $A(1) > r^*$*, the economy achieves sustained growth in the long run.*
2. *If* $A(1) < r^*$*, the economy reaches a steady state with* $L > 0$*.*
3. *If* $A(1) = r^*$*, then economy grows as long as* $L > 0$*. If* $L = 0$*, the economy stops growing at that point.*

# Existence of Steady State... (Corr. 1)

## Corollary

*(i) If* $a\,(i) > r^*$ *for all* $i$*, then* $A(1) > r^*$ *and there is sustained growth.*

*(ii) If* $\sigma < 1$ *and* $a\,(i) = 0$ *for a positive measure of tasks, then* $A\,(1) = 0 < r^*$ *and no long-run growth is possible.*

*(iii) If* $\sigma > 1$*, a sufficient condition for sustained growth is that there exists* $m$ *such that for all* $i \in [0, m]$ *we have* $a\,(i) \geq m^{\frac{1}{1-\sigma}} r^*$*.*

- $\sigma < 1$ means Labor is necessary for production.
- This plus no full automation is sufficient condition for steady state.
  - Not necessary.

# Implication for Automation

- Result implies technical progress can make qualitative difference.
- As long as $A(1) < r^*$, technological progress has "typical" results.
- But if $A(1)$ is pushed above $r^*$, reach different regime.
  - Sustained growth is possible through accumulation of capital.
  - Labor share goes to zero in long run.

# Special case: stepwise productivity

- Now let's focus on a special case: stepwise productivity.
- Suppose that $a\left(i\right)$ satisfies:

$$a\left(i\right)=\begin{cases}a & i\in\left[0,\bar{\alpha}\right]\\ 0 & i>\bar{\alpha}\end{cases}$$

- Labor productivity is $b=1$ for all $i$. Assume $a>r^{*}$.
- Now can cleanly distinguish two types of technological progress:
  - Traditional technical progress: increase in $a$.
  - Labor-substituting technical progress (automation): increase in $\bar{\alpha}$.

# Effects of Technical Progress

- Consider long run effects of technological progress.
  - Comparative statics of steady state.
- Traditional technological progress: Marginal increase in $a$ (Corr. 2):
  - Raises wage $w$.
  - Raises labor share if $\sigma < 1$; lowers $\sigma > 1$; constant $\sigma = 1$.
- Automation: Marginal increase in $\bar{\alpha}$ (Corr. 3):
  - Raises wage $w$.
  - Lowers labor share.

# Wage Decline

- Previous results hold for stepwise capital productivity.
  - Automation always raises wage.
- But does this always hold?
  - No! Possible for automation to **lower** worker wages, even in the long run.
- This never happens with constant worker task productivity $b(i)$.
  - Can happen when workers are more productive at tasks that get automated than remaining tasks.

# Other Results: Wage Decline

- For example, suppose capital and labor task productivity satisfy:

$$b\left(i\right)=\begin{cases}b_m & i\in\left[0,\bar{\alpha}_1\right]\\ b_1 & i\in\left(\bar{\alpha}_1,1\right]\end{cases}\qquad a\left(i\right)=\begin{cases}1 & i\in\left[0,\bar{\alpha}\right]\\ 0 & i>\bar{\alpha}\end{cases}$$

- Suppose initially we have $\bar{\alpha}=\bar{\alpha}_0<\bar{\alpha}_1$, and then $\bar{\alpha}$ increases to $\bar{\alpha}_1$.
- Steady state wage declines iff (Prop. 4):

$$\frac{b_m}{b_1}>\left[\frac{\left(a/r^*\right)^{1-\sigma}-\bar{\alpha}_1}{1-\bar{\alpha}_1}\right]^{\frac{1}{1-\sigma}}$$

where RHS is greater than 1 as $a>r^*$.

# Majority Voting

- Now suppose policy set by majority vote; workers in the majority.
- For simplicity, assume entrepreneurs have log utility:

$$u\left(c_e\right) = \log\left(c_e\right)$$

- Then entrepreneur consumption follows simple rule:

$$c_e = \rho K$$

- Suppose government spending is a fixed share of GDP:

$$G = \omega Y$$

- Substitute these into resource constraint to obtain:

$$\dot{K} = \left(1 - \omega\right) F\left(K, L\right) - \delta K - \rho K - C$$

# Planner's Problem

- Planner sets path of $\left\{\tau^{L}, \tau^{K}\right\}$ to maximize worker welfare:

$$\int_0^\infty e^{-\gamma t} U\left(C_w, L\right) dt$$

- Subject to constraint:

$$\dot{K} = (1-\omega) F\left(K, L\right) - \delta K - \rho K - C_w$$

- Plus non-negativity constraint on labor, $L \geq 0$.
- One state: $K$. Two controls: $\{C, L\}$.

# Optimality conditions

- Optimality conditions are:

$$\lambda = U_C(C_w, L)$$

$$-U_L(C_w, L) \leq \lambda(1-\omega) F_L(K, L)$$

$$-\frac{\dot{\lambda}}{\lambda} = (1-\omega) F_K(K, L) - \rho - \delta - \gamma$$

# Labor income tax

- We have labor condition:

$$-U_L(C_w, L) \leq (1-\omega)\, U_C(C_w, L) \cdot F_L(K, L)$$

- Compare with equilibrium condition:

$$-U_L(C_w, L) \leq \left(1-\tau^{\ell}\right) U_C(C_w, L) \cdot F_L(K, L)$$

- This implies constant labor income tax:

$$\tau^{\ell} = \omega$$

- If $\omega = 0$ (no government spending), then zero labor income tax.

# Capital Tax

- The expressions above imply that optimal capital taxation satisfies:

$$1 - \tau^k = (1 - \omega) \left( \frac{\dot{K}/K + \delta + \rho}{\rho + \gamma + \delta - \dot{\lambda}/\lambda} \right)$$

- In steady state:

$$\tau^k = \frac{\gamma + \omega (\rho + \delta)}{\rho + \delta + \gamma} = \omega + \frac{(1 - \omega)\gamma}{\rho + \delta + \gamma} > \omega = \tau^\ell$$

  - Independent of technology. Always positive and larger than labor tax.

- Away from steady state, decreasing in $\dot{K}/K$, increasing in $\dot{C}/C \propto -\dot{\lambda}/\lambda$.

# Existence of Steady State vs. Sustained Growth (Prop. 5)

- A steady state exists as long as:

$$A(1) < \frac{\rho + \delta + \gamma}{1 - \omega} = r^*$$

- Sustained growth through accumulation of capital occurs when:

$$A(1) > \frac{\rho + \delta + \gamma}{1 - \omega} = r^*$$

- (Equals is a knife-edge case. Growth rate approaches $0$ asymptotically.)
- Note that the steady state capital tax rate can be written as:

$$\tau_{ss}^k = \omega + \frac{\gamma}{r^*}$$

# Growth with CRRA utility

- Suppose $A(1) > \frac{\rho+\delta+\gamma}{1-\omega}$ so there is sustained growth.
- Suppose workers have CRRA ($\varphi$) utility.
- Balanced growth path (BGP) exists if:

$$\varphi > 1 - \frac{\gamma}{(1-\omega)A(1) - \delta - \rho}$$

  - Condition always holds when $\varphi \geq 1$.

# Balanced Growth Path (Prop. 5)

- Growth rate on BGP will be:

$$g = \frac{(1-\omega)\,A\,(1) - \delta - \rho - \gamma}{\varphi} > 0$$

- Capital tax rate on BGP is:

$$\tau^k_{bgp} = \omega + \frac{(\varphi - 1)\,g + \gamma}{A\,(1)}$$

- If log utility ($\varphi = 1$):

$$\tau^k_{bgp} = \omega + \frac{\gamma}{A\,(1)}$$

# Robot Taxes

- Is it ever optimal to tax robots specifically?
  - Suppose we partition tasks into two types: 1 and 2.
  - Can set different tax rates on capital income from each type.
- Then can express production function as

$$F(K_1, K_2, L)$$

- Production satisfies:

$$\left(1 - \tau_1^k\right) F_{K_1} = \left(1 - \tau_2^k\right) F_{K_2}$$

- By varying tax rates, planner can effectively choose both $K_1$ and $K_2$.

# Robot Taxes

- Planning problem: maximize worker welfare subject to:

$$\dot{K} = F(K - K_2, K_2, L) - \delta K - \rho K - C_w$$

- $K$ is state. $(L, C, K_2)$ are choice variables. $K_2 \in [0, K]$.
- FOC wrt $K_2$ gives us:

$$F_{K_1} = F_{K_2}$$

  unless non-negativity constraints bind.
- Implies $\tau_1^k = \tau_2^k$. No robot taxes.
- Result generalizes to arbitrary partitions of tasks.

# Quantitative Exercise

- Now we will look at a quantitative analysis of an episode of automation.
- Workers have log log utility: $U\left(C,L\right)=\log\left(C\right)+\phi\log\left(1-L\right)$
- Piecewise technology:

$$a\left(i\right)=\begin{cases} a & i\in\left[0,\bar{\alpha}\right] \\ 0 & i>\bar{\alpha} \end{cases}$$

# Quantitative Exercise

| $\rho$ | $\gamma$ | $\delta$ | $\omega$ | $\phi$ | $a$ | $b$ | $\bar{\alpha}$ | $\tau^k$ | $\tau^\ell$ |
|---|---|---|---|---|---|---|---|---|---|
| $0.04$ | $0.06$ | $0.1$ | $0.11$ | $1.4$ | $0.5$ | $1$ | $0.5(0.25)$ | $0.36$ | $0.23$ |

Table: Calibration

- Consider two values of CES across tasks: $\sigma = 0.8$ and $\sigma = 1.2$.
- Initial $\bar{\alpha}$ is $0.5$ for $\sigma = 0.8$, and $0.25$ for $\sigma = 1.2$.
- Steady state exists under these parameters.
- Gradual increase in $\bar{\alpha}$, calibrated to double steady state output.
- Consider under both a fixed tax regime, and under majority voting.

| | Initial ($\sigma = 0.8$) | Auto. $(0.8)$ | Initial $(1.2)$ | Auto. $(1.2)$ |
|---|---|---|---|---|
| $Y$ | 1.195 | 2.389 | 2.159 | 4.318 |
| $K/Y$ | 1.937 | 2.506 | 1.929 | 2.398 |
| $C_w/Y$ | 0.619 | 0.539 | 0.620 | 0.554 |
| $c_e/Y$ | 0.077 | 0.100 | 0.077 | 0.096 |
| $wL/Y$ | 0.576 | 0.452 | 0.578 | 0.475 |
| $rK/Y$ | 0.424 | 0.548 | 0.422 | 0.525 |
| $T_w/Y$ | 0.175 | 0.186 | 0.175 | 0.186 |
| $T_w/Y$ *adj* | 0.106 | 0.137 | 0.106 | 0.131 |
| $\bar{\alpha}$ | 0.500 | 0.647 | 0.250 | 0.311 |
| $\tau^k$ | 0.360 | 0.360 | 0.360 | 0.360 |
| $\tau^\ell$ | 0.230 | 0.230 | 0.230 | 0.230 |

Table: Steady states before and after automation with fixed taxes.

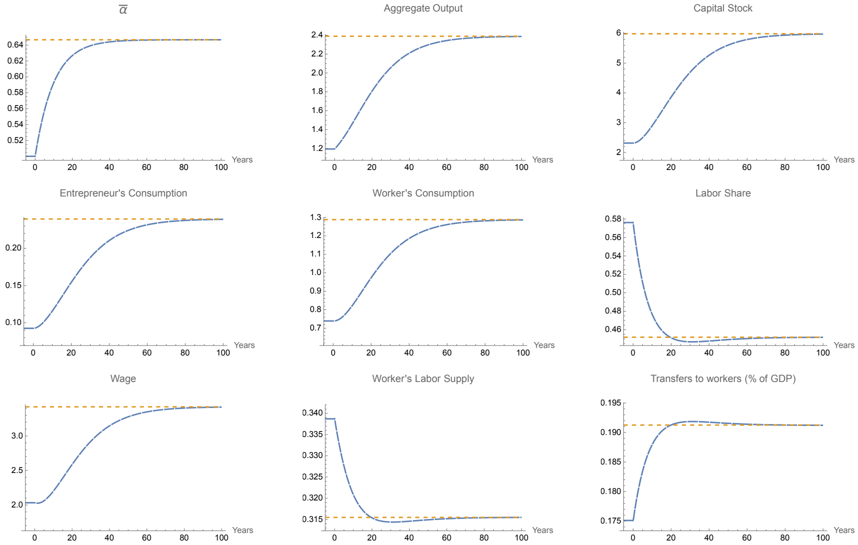


Figure: Automation episode with fixed tax rates and $\sigma = 0.8$.

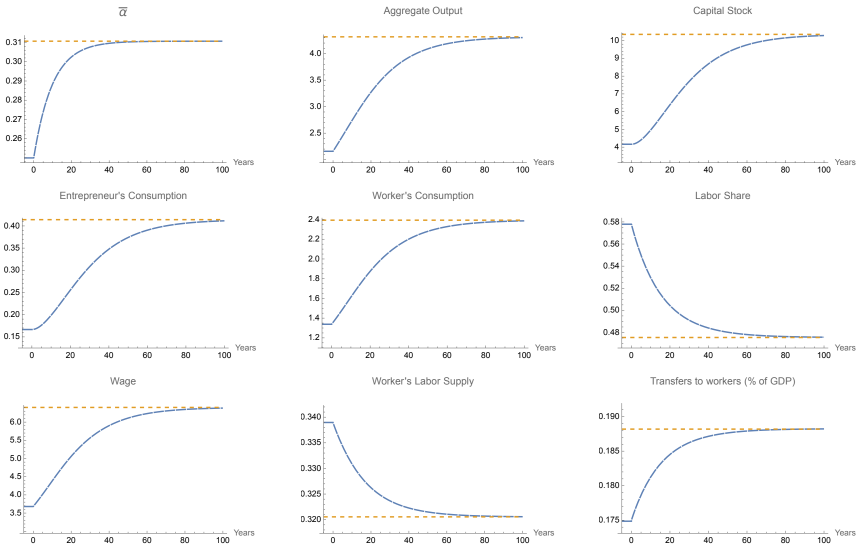


Figure: Automation episode with fixed tax rates and $\sigma = 1.2$.

# Majority Voting

- Now consider the same episode of automation under majority voting.
- Start at same steady state as before
- Then **two** things happen at the same time:
  - An episode of automation (same as before).
  - Policy starts to follow majority voting.

| | Initial ($\sigma = 0.8$) | Auto. ($0.8$) | Auto. + M.V. ($0.8$) |
|---|---|---|---|
| $Y$ | 1.195 | 2.389 | 2.530 |
| $K/Y$ | 1.937 | 2.506 | 2.452 |
| $C_w/Y$ | 0.619 | 0.539 | 0.547 |
| $c_e/Y$ | 0.077 | 0.100 | 0.098 |
| $wL/Y$ | 0.576 | 0.452 | 0.449 |
| $rK/Y$ | 0.424 | 0.548 | 0.551 |
| $T_w/Y$ | 0.175 | 0.186 | 0.147 |
| $T_w/Y$ *adj* | 0.106 | 0.137 | 0.147 |
| $\bar{\alpha}$ | 0.500 | 0.647 | 0.647 |
| $\tau^k$ | 0.360 | 0.360 | 0.377 |
| $\tau^\ell$ | 0.230 | 0.230 | 0.110 |

Table: Steady states: automation + majority voting

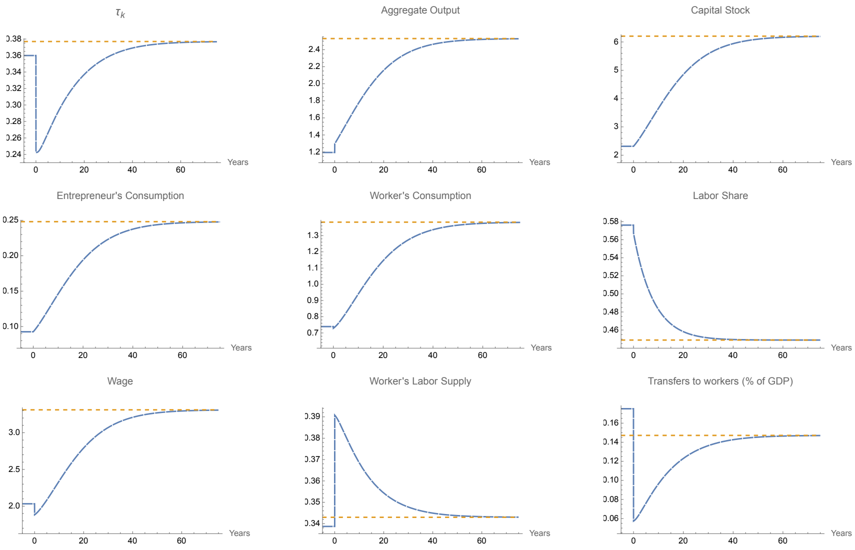


Figure: Automation episode under majority voting ($\sigma = 0.8$)

| | Initial ($\sigma = 1.2$) | Auto. (1.2) | Auto. + M.V. (1.2) |
|---|---|---|---|
| $Y$ | 2.159 | 4.318 | 4.548 |
| $K/Y$ | 1.929 | 2.398 | 2.322 |
| $C_w/Y$ | 0.620 | 0.554 | 0.565 |
| $c_e/Y$ | 0.077 | 0.096 | 0.093 |
| $wL/Y$ | 0.578 | 0.475 | 0.478 |
| $rK/Y$ | 0.422 | 0.525 | 0.522 |
| $T_w/Y$ | 0.175 | 0.186 | 0.139 |
| $T_w/Y$ *adj* | 0.106 | 0.131 | 0.139 |
| $\bar{\alpha}$ | 0.250 | 0.311 | 0.311 |
| $\tau^k$ | 0.360 | 0.360 | 0.377 |
| $\tau^\ell$ | 0.230 | 0.230 | 0.110 |

Table: Steady states: automation + majority voting

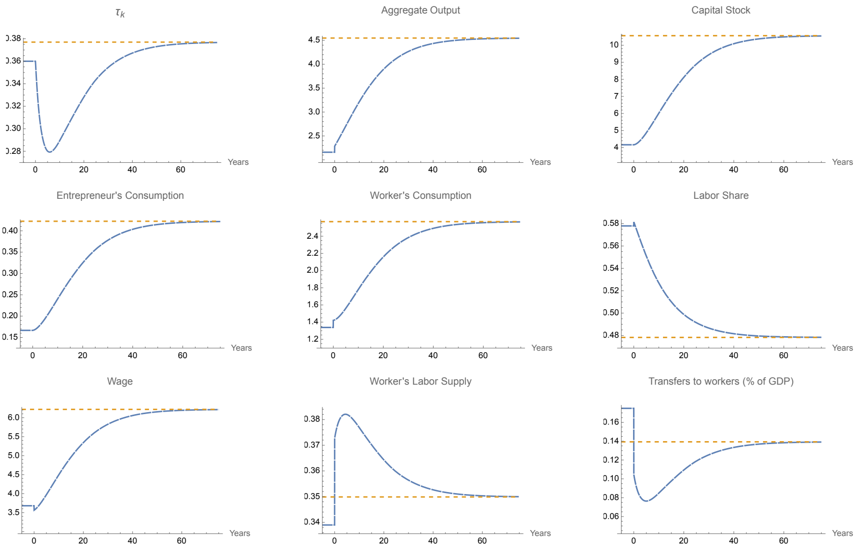


Figure: Automation episode under majority voting ($\sigma = 1.2$)

# Welfare Gains from automation

- Let's look at welfare gains from automation.
  - Calculated in consumption equivalent terms.
- Welfare gains from automation ($\sigma = 0.8$):
  - Fixed taxes: $24.6\%$ for workers; $57.7\%$ for entrepreneurs.
  - Majority voting: $26.1\%$ for workers; $84.5\%$ for entrepreneurs.
- Welfare gains from automation ($\sigma = 1.2$):
  - Fixed taxes: $29.4\%$ for workers; $49.0\%$ for entrepreneurs.
  - Majority voting: $30.6\%$ for workers; $72.5\%$ for entrepreneurs.

# Welfare Gains from automation

- Observations:
  - Significant welfare gains for both workers and entrepreneurs.
  - However, gains proportionally greater for entrepreneurs.
  - Majority voting increases welfare gains for both.
  - However, entrepreneurs benefit a lot more.
- Counterintuitive!
  - Majority voting is set to maximize worker welfare only.
  - Yet entrepreneurs end up benefiting more!
- Intuition:
  - Optimal to lower capital taxes during transition.
  - Benefits workers a little, entrepreneurs a lot.

# Conclusions

- Automation differs from traditional technological progress:
  - Can cause long-run sustained growth.
  - Lowers labor share, raises wages (in piecewise case).
  - Effect depends on $\sigma$.
- When workers have political power and there is a UBI:
  - Long run capital tax independent of automation/technology.
  - Transfers increase with automation in absolute and relative terms.
  - Lower capital taxes during automation episode, higher in long run.
- Both workers and entrepreneurs benefit from majority voting policy.
  - But entrepreneurs benefit more.