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Thermal Conduction and the Rayleigh-Taylor Instability

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Abstract : Hydrodynamic instabilities are of critical importance in several basic physics and engineering applications. Here we are concerned with inertial confinement fusion (ICF). The ICF is one of the potential solutions to solve growing energy demands of the world and, at the same time, thanks to being essentially pollution-free, help taming some negative trends observed in Earth natural environment (global warming). In the context of ICF technology, hydrodynamic instabilities place fundamental limitations on the experimental design parameters that affect capsule ignition. Thus, understanding these instabilities is crucial to improving the performance and energy gains achievable by ICF. The hydrodynamic instability of interest in this work is the Rayleigh-Taylor (RT) instability since this is the dominant instability during the ICF capsule compression and leads to shell disruption and overall target performance degradation. Much work has been done on understanding RT instabilities in pure hydrodynamic experiments and in simulations. In ICF however there are other types of physics besides hydrodynamics at work and it is well known that pure hydro simulations fail to account for the way in which RT instabilities form and propagate in actual ICF experiments. One type of physics known to be present in ICF experiments is thermal conduction. The focus of this work is to investigate the effects of thermal conduction on RT instabilities. Although general physical properties of plasma in that regime are generally well-known, we lack information about more specific characteristics. One such property is efficiency of the thermal conduction, which is expressed by a coefficient of thermal conductivity. Furthermore, the conditions relevant to ICF can be obtained only in the course of unique ICF-like (high-energy density) experiments. Such experiments are very expensive making it a challenge to be able to accurately measure the non-constant thermal conductivity coefficient accurately. In this work we explore a technique for measuring this coefficient with the goal of understanding the effect of thermal conduction on RT instabilities and validating our simulation codes against ICF experimental data.

Inertial Confinement Fusion, Clean Energy

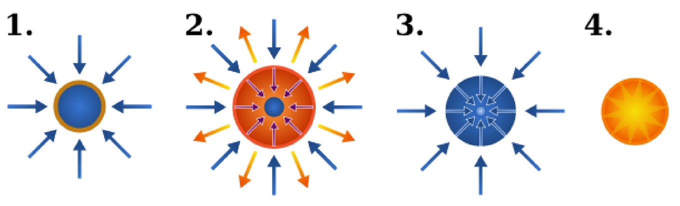

Figure 1: (The ICF Process) 1) Lasers heat the surface of the target. 2) Surface material is ablated causing the fuel core to implode. 3) Final stage of fuel implosion, very high density fuel ignites. 4) Thermonuclear burn front propagates outward through compressed fuel causing ignition and producing much more energy than what was needed to initiate the process.

Inertial confinement fusion (ICF) is an approach to generating fusion which relies on the inertia of the fuel mass for confinement.

In general ICF systems use a high powered laser, the driver, whose beam is split up into a number of beams. These are delivered into a chamber (called a target chamber) by a number of mirrors, positioned in order to illuminate the target evenly over its whole surface, or perhaps a portion of the surface during experimental exercises. The heat applied by the laser causes the outer layer of the target to explode, the ablation phase.

The material exploding off the surface causes the remaining material on the inside to be driven inwards eventually collapsing into a tiny spherical ball, or close to spherical. In ICF the density of the resulting fuel mixture is very large in magnitude. However, the density is not large enough to initiate the desired rate of fusion by itself. However, while the central fuel is imploding, shock waves form and propagate into the center of the fuel. When the shock waves meet near the center of the compressed fuel, the density of that spot becomes much larger.

Assuming appropriate conditions, the fusion rate in the region highly compressed by the shock wave can give off significant amounts of alpha particles, which are highly energetic. Since the density of the surrounding fuel is large, the alpha particles move only a short distance before losing their energy as heat to the fuel. This added heat energy will cause more fusion reactions which will then produce more alpha particles, thus initiating a chain reaction. The fusion process spreads outward leading to a self sustaining burn, the ignition.

Hydrodynamic instabilities are of great importance to achieving efficient ICF because they relate directly to the efficiency and gain of the ICF process. When there is an uneven compression of the central fuel then the interface between the cold and hot fuel will be perturbed thus forming Rayleigh-Taylor (RT) fluid instabilities. This fluid instability induces mixing between the hot and cold fuel. This mixing can reduce the heat efficacy at the time of maximum compression of the fuel which in turn directly effects the gain of the fusion process and thus its efficiency as well. Investigating the Rayleigh-Taylor hydrodynamic instability, in particular, within the context of ICF will be the primary focus of this work.

Rayleigh-Taylor Instability

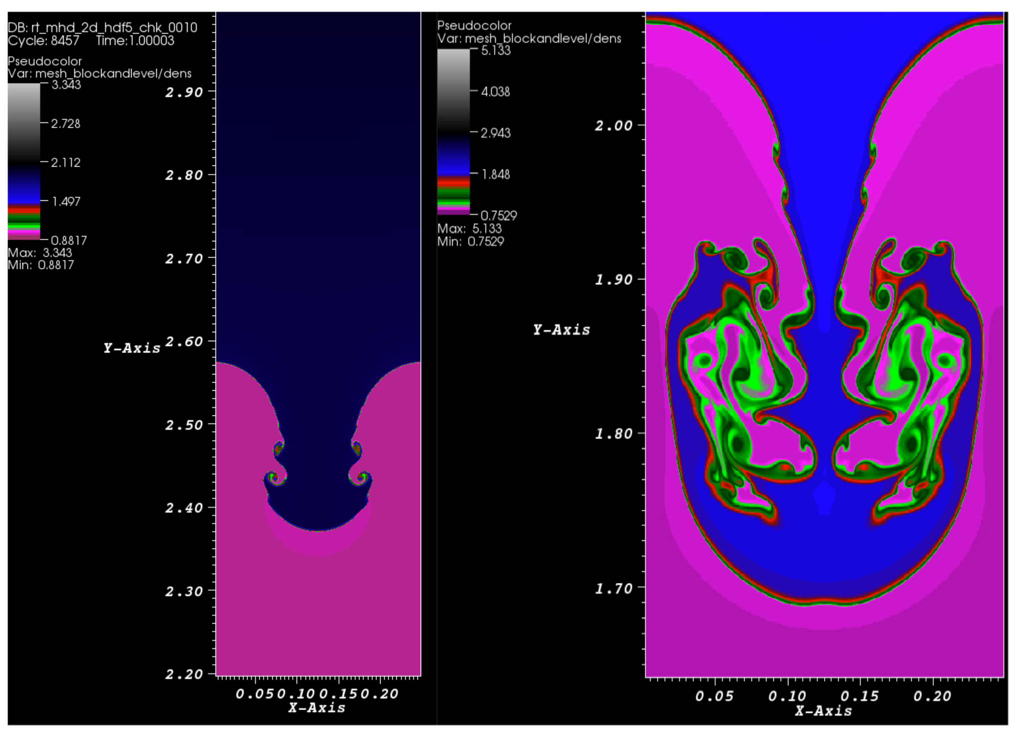

Figure 2: RTI in the Linear and Nonlinear Regime - The Density Field.

The Rayleigh-Taylor instability is an instability that initiates when the interface between two fluids of different densities becomes subjected to a normal pressure gradient with direction such that the pressure is higher in the lighter fluid than in the more dense fluid. An example of the type of partial differential equation where this phenomenon can occur is in hyperbolic systems such as the Euler-Lagrange equations.

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_j}(\rho v_j) = 0 \tag{1}$$

$$\frac{\partial}{\partial t}(\rho v_i) + \frac{\partial}{\partial x_j}(\rho v_i v_j) = -\frac{\partial p}{\partial x_i} + \rho F_i \quad (i = 1,2,3) \tag{2}$$

$$\frac{\partial(\rho E)}{\partial t} + \frac{\partial}{\partial x_j}((\rho E + p)v_j) = 0 \tag{3}$$

where ρ is the density, p is the pressure, E is the total energy, v are the components of velocity, and F is some external force such as gravity, for example. The first equation represents the conservation of mass, the second conservation of momentum, and the third conservation of energy.

In this linear regime the growth of a small perturbation at the interface is exponential in time [1]. The growth rate of this perturbation is given by,

$$\tau_{RT} = e^{\sqrt{\mathcal{A} g \omega} t} \tag{4}$$

where, g is the effective gravitational constant, ω is the wave number, and $\mathcal{A}$ is called the Atwood number. The Atwood number is given by, $\mathcal{A} = \frac{\rho_{heavy} - \rho_{light}}{\rho_{heavy} + \rho_{light}}$. When the amplitude of the initial perturbations grows to sizes on the order of ten to forty percent of the initial wavelength then substantial deviations from the linear theory become observable.

Effect of Thermal Conduction on Rayleigh-Taylor Instability

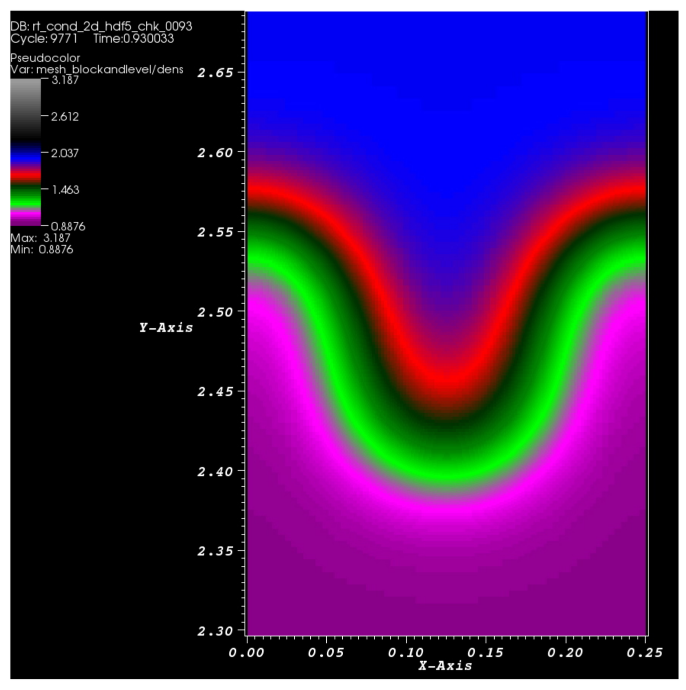

Figure 3: RTI with Constant Thermal Conduction in Nonlinear Regime - The Density Field.

Early work done by Youngs [3], in 1984, on using numerical simulations to understand turbulent mixing by Rayleigh-Taylor instabilities in the context of ICF experiments has already made clear that pure hydrodynamical simulations of RTI mixing is not enough to account for what is occurring in actual ICF experiments. It has been known, even before Youngs work, that there are other physical processes interacting with the RTI propagation in actual ICF experiments. There are radiative transport, magnetic field interactions, and thermal conduction processes involved in the actual experiments, for example. Multiphysics simulations are costly and so much of the research has continued to be driven by pure hydro based simulations. We couple the fluid equations with thermal conductivity by solving a heat equation and adjusting the energy equation as follows,

$$\frac{\partial(\rho E)}{\partial t} + \frac{\partial}{\partial x_j}((\rho E + p)v_j) + \nabla \cdot (\kappa \nabla T) = 0. \tag{5}$$

In comparing FIG(2b) and FIG(3), both of which were allowed to evolve well into the nonlinear regime, one can note that even with this simplified form of constant thermal conductivity it has had a very significant effect on the evolved structure of the instability. To accurately account for thermal conductivity we must have a method to measure the thermal conductivity coefficient in a an ionized plasma, which is non-constant.

Measuring Thermal Conductivity

In [2], Ryutov points towards a technique for taking measurements of the thermal conduction coefficient experimentally. Ryutov points out that in the laboratory one could create a situation where the classical RT perturbations are stable whereas those driven by thermal conductivity are unstable. By embedding a known perturbation in the ICF design target one can observe it's growth in a standard way, through a laser driven experiment. As he has shown, that instabilities exist only with finite thermal conductivity, then the growth rate will be a direct measure of the thermal conductivity coefficient through the use of the dispersion relation:

$$\frac{1}{\Gamma^2}\frac{gk^2}{k^2+q^2}\left(\frac{\rho'}{\rho} + \frac{g}{s_{AD}^2}\right) - 1 = \frac{\chi^2(k^2+q^2)}{s_{AD}^2 \Gamma}\left(1 + \frac{p_C}{s_{IS}^2 \rho}\frac{gC'}{\Gamma^2}\frac{k^2}{k^2+q^2}\right) \tag{6}$$

where s_{AD} and s_{IS} are an adiabatic and isothermal sound speeds, g is the acceleration, C is the compositional constant, and Γ is the complex RT growth rate. Instead of thermal conductivity κ we have introduced the thermal diffusivity χ,

$$\chi = \frac{\kappa}{\varepsilon_T}. \tag{7}$$

The dispersion relation given by Eq. (6) involves quantities either known analytically or that can be calculated throughout an experiment or simulation leaving the only unknown to be the thermal diffusivity, thus giving us a direct method to calculate the thermal conductivity of the material medium with which we are working.

If we consider (6) in the limit as the thermal diffusivity χ goes to zero, one is able to recover the dispersion relation for adiabatic perturbations,

$$\Gamma^2 = \Gamma_{AD}^2 \equiv \frac{gk^2}{k^2+q^2}\left(\frac{\rho'}{\rho} + \frac{g}{s_{AD}^2}\right). \tag{8}$$

On the other hand, if we consider the limiting case of very high thermal diffusivity we recover a different class of perturbations, in the isothermal regime, who's stability is determined by the sign of the product $p_C C'$:

$$\Gamma^2 = \Gamma_{IS}^2 \equiv -\frac{gk^2}{k^2+q^2}\frac{p_C C'}{\rho s_{IS}^2}. \tag{9}$$

The quantities Γ_{AD} and Γ_{IS} represent growth rates in the adiabatic and isothermal regimes respectively. In general, the quantity Γ which is determined via (6) is not strictly within either an adiabatic or an isothermal regime. This dispersion relation shows us that in the adiabatic regime we recover the growth rate of a pure hydro or classic Rayleigh-Taylor instability. When the thermal diffusivity is not negligible then the quantity Γ ceases to behave like the classic RTI. This is because the effect of thermal conduction on the fluid changes the pure hydro instabilities significantly so that what we observe is the propagation of an entirely new type of instability.

References

[1] S. Chandrasekhar, *Hydrodynamic and Hydromagnetic Stability* , Dover Publications, New York, 1981

[2] Dmitri D. Ryutov, *Destabilizing effect of thermal conductivity on the Rayleigh-Taylor instability in a plasma* , Phys. Plasmas, Vol 7, 4797, September 2000

[3] David L. Youngs, *Numerical Simulation of Turbulent Mixing by Rayleigh-Taylor Instability.* ,Physica 12D, pp 32-44, 1984