Discovery of Governing Equations in Wildfire Spread Using Weak SINDy

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Background

Wildfires are one of the most dynamic and complex natural phenomena, affecting ecosystems, economies, and human safety across the globe. Understanding the mechanisms driving wildfire spread is essential for improving prediction models, which can aid in the management and mitigation of these devastating events. Traditional approaches to modeling wildfire behavior often rely on empirical formulas or computational fluid dynamics simulations, which, while useful, can fall short in capturing the intricate, nonlinear dynamics of fire spread in real-world conditions.

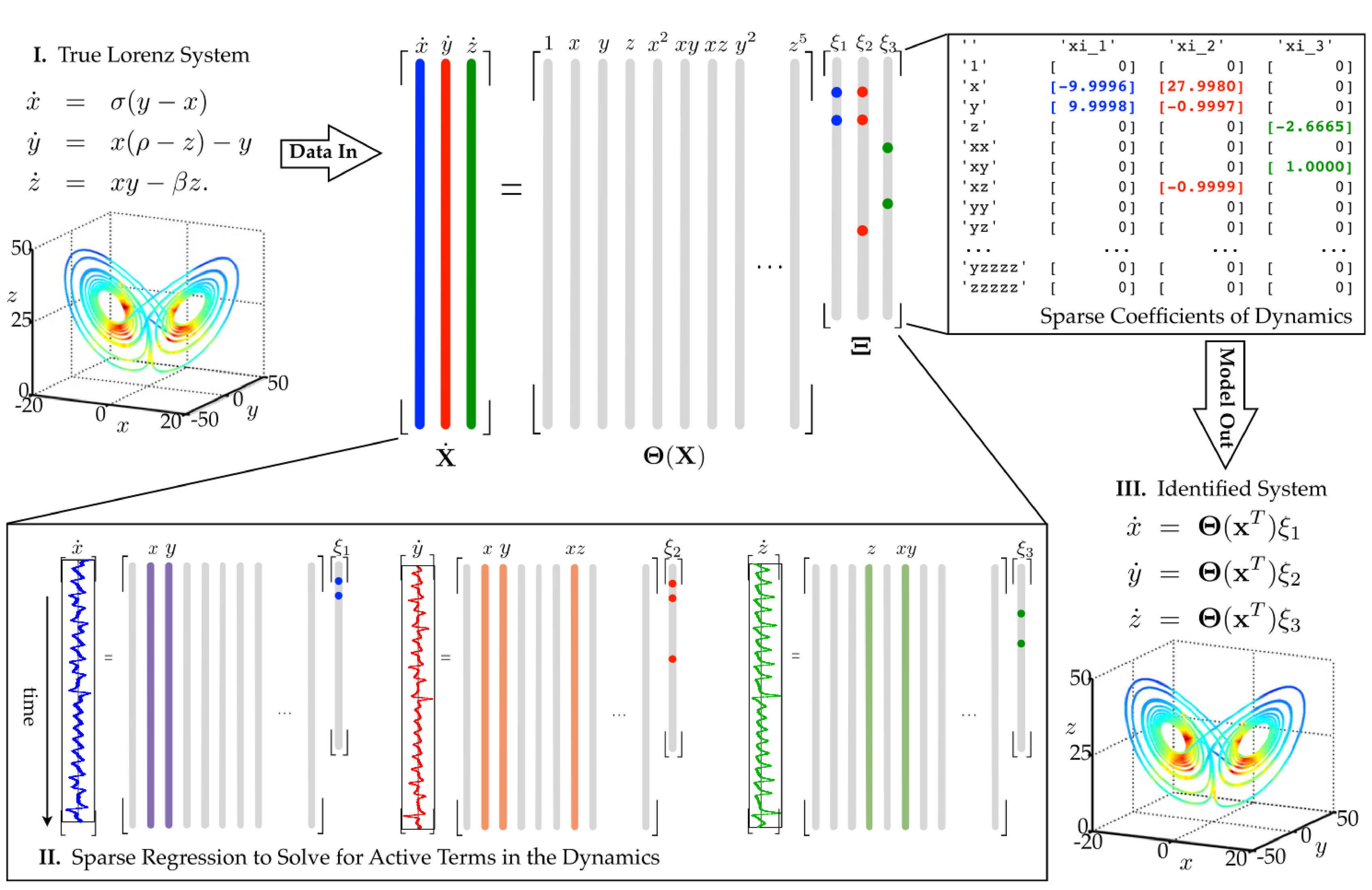
Enter the Sparse Identification of Nonlinear Dynamical systems (SINDy) algorithm, a powerful tool in the data-driven discovery of dynamical systems. The weak formulation of SINDy (weak SINDy) extends this capability, allowing for the identification of governing equations from data that may be noisy, sparse, or otherwise challenging to analyze using traditional methods. This project aims to leverage the weak SINDy method to identify the governing equations of fire spread, using detailed empirical data from a controlled head fire spreading across a 2m x 2m plot of pine straw, sampled every second.

Pyrocumulus clouds of the West Fork Complex Fire, June 2013.

By focusing on this specific scenario, the study seeks to provide novel insights into the fundamental processes that dictate fire spread, bridging the gap between empirical observations and theoretical modeling. The outcome has the potential to significantly enhance our understanding of wildfire dynamics, offering a pathway toward more accurate and predictive models that can inform fire management strategies and emergency response efforts.

Weak SINDy Overview

The Sparse Identification of Nonlinear Dynamical Systems (SINDy) algorithm is a revolutionary approach that uses sparse regression to discover the underlying differential equations from data. The "weak" formulation of SINDy, known as weak SINDy, further enhances this method by integrating the data into a set of weak form equations. This is particularly useful when dealing with noisy, incomplete, or indirectly observed data, common challenges in environmental and experimental settings.



Advantages

- **Handling Noisy and Sparse Data** - Weak SINDy is robust to noisy, complex, and turbulent behaviors, efficiently filtering out noise and focusing on capturing the core dynamics of the system.
- **Unveiling Nonlinear Dynamics** - Weak SINDy excels in identifying nonlinear relationships and interactions within wildfire data, such as wind, humidity, and fuel type, providing a clear mathematical formulation of these dynamics.
- **Model Simplicity and Interpretability** - One of the strengths of weak SINDy is its ability to produce sparse, interpretable models that are easy to analyze and understand.
- **Data-Driven Discovery** - Weak SINDy offers a pathway to derive more accurate and validated models directly from observational data.
- **Scalability and Flexibility** - Weak SINDy can be adapted to various scales and types of data, making it suitable for expanding beyond the initial experimental setup to larger, more heterogeneous landscapes.

References

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- Messenger, D. A., & Bortz, D. M. (2021). Weak SINDy for partial differential equations. Journal of Computational Physics, 443, 110525.
- Sagel, D., Speer, K., Pokswinski, S., & Quaife, B. (2021). Fine-Scale Fire Spread in Pine Straw. Fire, 4(4), 69.

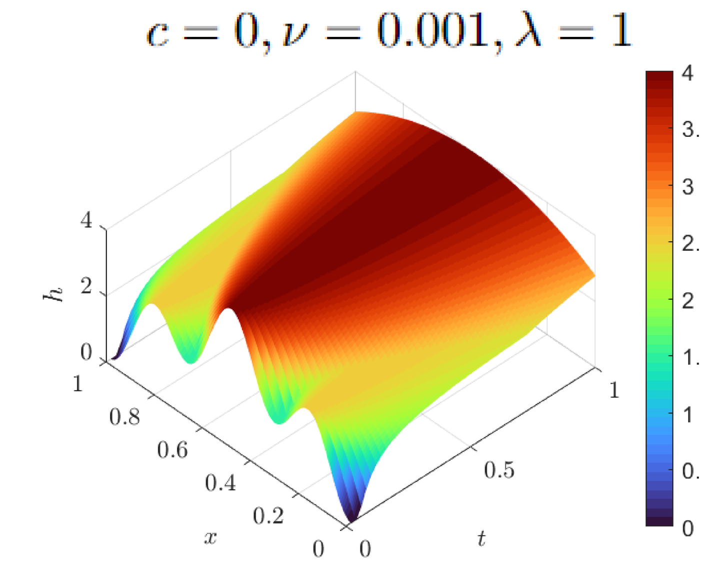
Results

The Kardar-Parisi-Zhang (KPZ) equation is a non-linear PDE that models the time-dependent height field of moving interfaces

$$\frac{\partial h}{\partial t} = \underbrace{c}_{\text{general upward motion}} + \underbrace{\frac{\lambda}{2}(h_x)^2}_{\text{non-linear advection}} + \underbrace{\nu h_{xx}}_{\text{linear diffusion}}$$

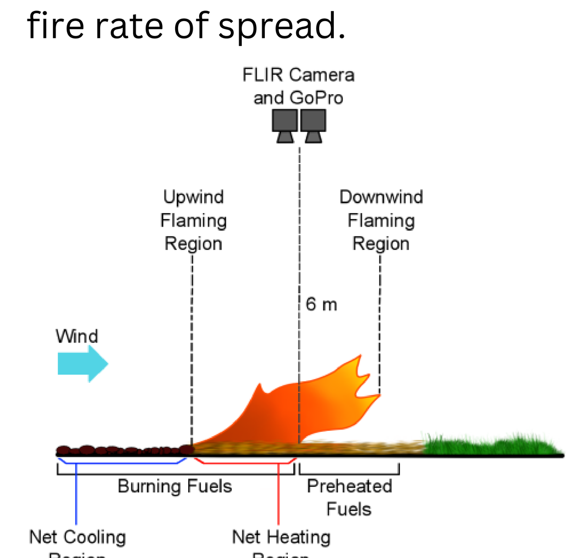
Synthetic data was created by William Callender, an undergraduate who uses a finite difference forward solver for the KPZ equation, combined with an optimization technique, to estimate the parameters in the KPZ equation.

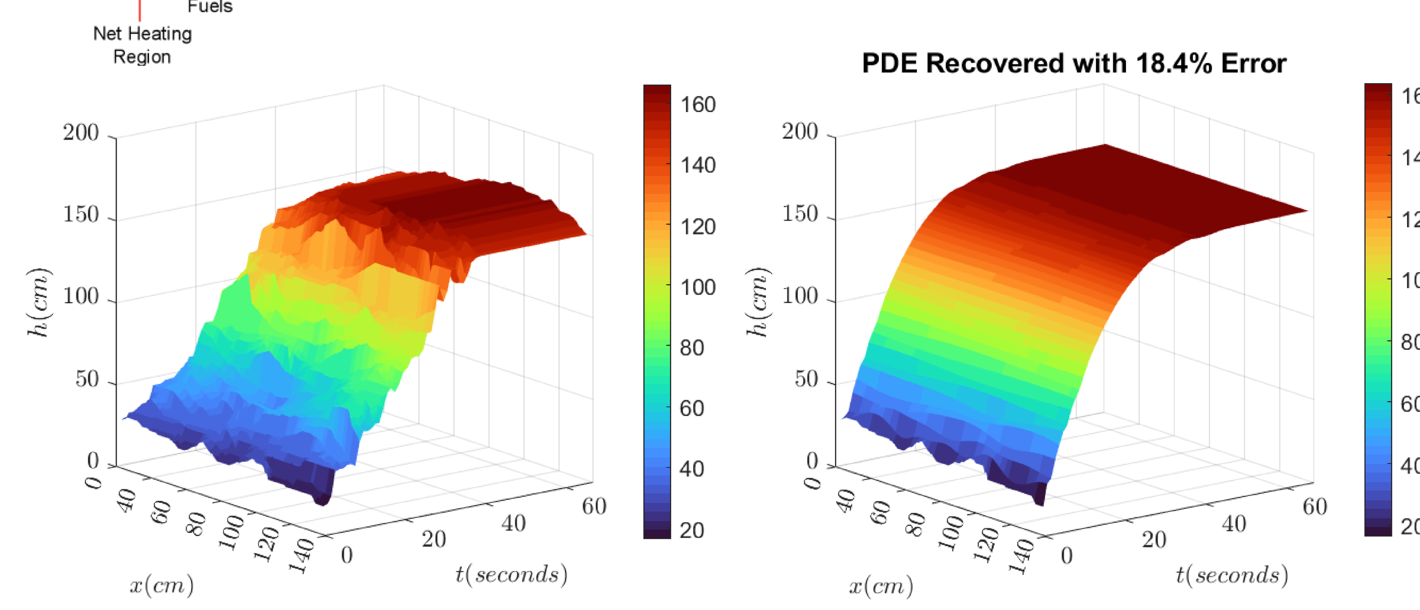
$$\partial_t h = 2 - \cos(6\pi x) - \cos(2\pi x)$$
$$c = 0, \nu = 0.001, \lambda = 1$$



$$\partial_t h = -5.06 + 3.13h - 0.1\partial_{xx}(h) - 0.4h^2 + 0.02\partial_{xx}(h^2)$$

Daryn Sagel tracked the location of the upwind flaming region using segmentation, cleaning, and edge detection applied to infrared and visual data and used to generate statistical characterizations of the fire rate of spread.





$$\partial_t h = \underbrace{\boxed{4.6}}_{\text{mean rate of spread}} + \underbrace{\boxed{0.07h - 0.0006h^2}}_{\text{Bernouilli equation}} + \underbrace{\boxed{8\partial_x(h)}}_{\text{linear advection}} - \underbrace{\boxed{0.03\partial_x(h^2)}}_{\text{nonlinear Burgers's advection}}$$

Preliminary analyses reveals distinct nonlinear patterns and interactions within the fire spread dynamics, which traditional models have not captured. The identified governing equations highlight the significant role of specific physical parameters and conditions in shaping fire behavior. These findings underscore the potential of weak SINDy in offering a more nuanced understanding of wildfire spread mechanisms.

The application of weak SINDy to fire spread data represents a promising step forward in the predictive modeling of wildfires. By uncovering the fundamental equations governing fire dynamics, this method opens new avenues for the development of accurate, efficient, and interpretable models.

Future Work

Future work will focus on uncovering the fundamental equations governing atmosphere-fire dynamics from recent experiments conducted firstly, to investigate the intricate atmosphere-fire interactions surrounding a wildland fire, with a particular emphasis on how varying slope angles and fuel loads influence these dynamics; and secondly, to identify the critical slope angle at which flame attachment occurs, marking a significant increase in fire spread potential.

0 degree slope

15 degree slope

30 degree slope